Collider Physics
Lecture V: Heavy quark production and decay

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Slides available from http://theory.fnal.gov/people/ellis/Talks/TASI06/

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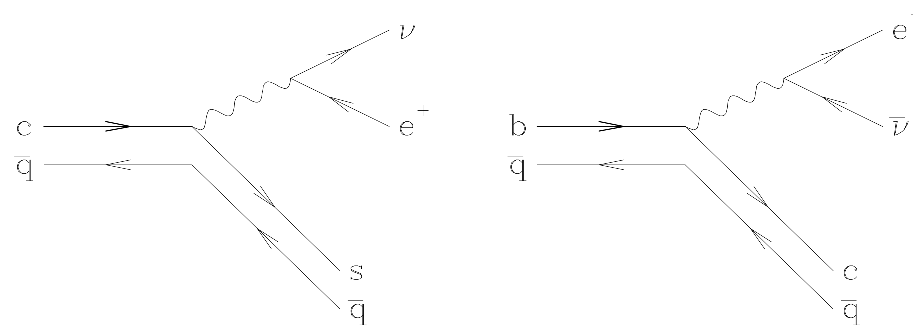
Charm and bottom quark decays

- treat the semi-leptonic decays of hadrons containing c and b quarks in analogy with the decay of a free muon, (*spectator model*)
- Lagrangians for CKM-favoured decays are

$$\mathcal{L}^{(c)} = -\frac{G_F}{\sqrt{2}} V_{cs}\, \bar{s}\gamma^\mu(1-\gamma_5)c\, \bar{\nu}\gamma_\mu(1-\gamma_5)e\ ,$$

$$\mathcal{L}^{(b)} = -\frac{G_F}{\sqrt{2}} V_{cb}\, \bar{c}\gamma^\mu(1-\gamma_5)b\, \bar{e}\gamma_\mu(1-\gamma_5)\nu\ .$$



$$\overline{\sum}|\mathcal{M}^{(c)}|^2 = 64\, G_F^2\, |V_{cs}|^2\, c\cdot e^+\ s\cdot\nu\ ,$$

$$\overline{\sum}|\mathcal{M}^{(b)}|^2 = 64\, G_F^2\, |V_{cb}|^2\, b\cdot\bar{\nu}\ c\cdot e^-\ ,$$

where $b, c, s, \nu, \bar{\nu}, e^+$ and e^- now stand for the four-momenta of the particles in the decay.

- by angular momentum conservation s and ν momenta prefer to be anti-parallel in c quark decay. The endpoint configuration in which the e^+ recoils against the parallel s and ν is thus disfavoured. We expect a soft spectrum for the positron.
- Conversely we expect a hard spectrum for the neutrino (or for the electron coming from the decay of a b quark).

$$\Gamma_{\text{sl}}^{(Q)} = \frac{m_Q}{2^8\pi^3} \int dx dy\, \theta(x+y-x_m)\, \theta(x_m - x - y + xy) \overline{\sum} |\mathcal{M}^{(Q)}|^2$$

- x and y are the rescaled energies of the charged and neutral leptons, $x = 2E_e/m_Q$, $y = 2E_\nu/m_Q$ in the frame in which the heavy quark Q is at rest. The kinematic endpoint of the spectrum is denoted by x_m and is given by $x_m = 1 - \epsilon^2$ where $\epsilon = m_q/m_Q$. The result for the semi-leptonic widths of the c and the b is

$$\frac{d\Gamma_{\text{sl}}^{(c)}}{dxdy} = |V_{cs}|^2 \Gamma_0(m_c)[12x(x_m - x)]$$

$$\frac{d\Gamma_{\text{sl}}^{(b)}}{dxdy} = |V_{cb}|^2 \Gamma_0(m_b)[12y(x_m - y)]$$

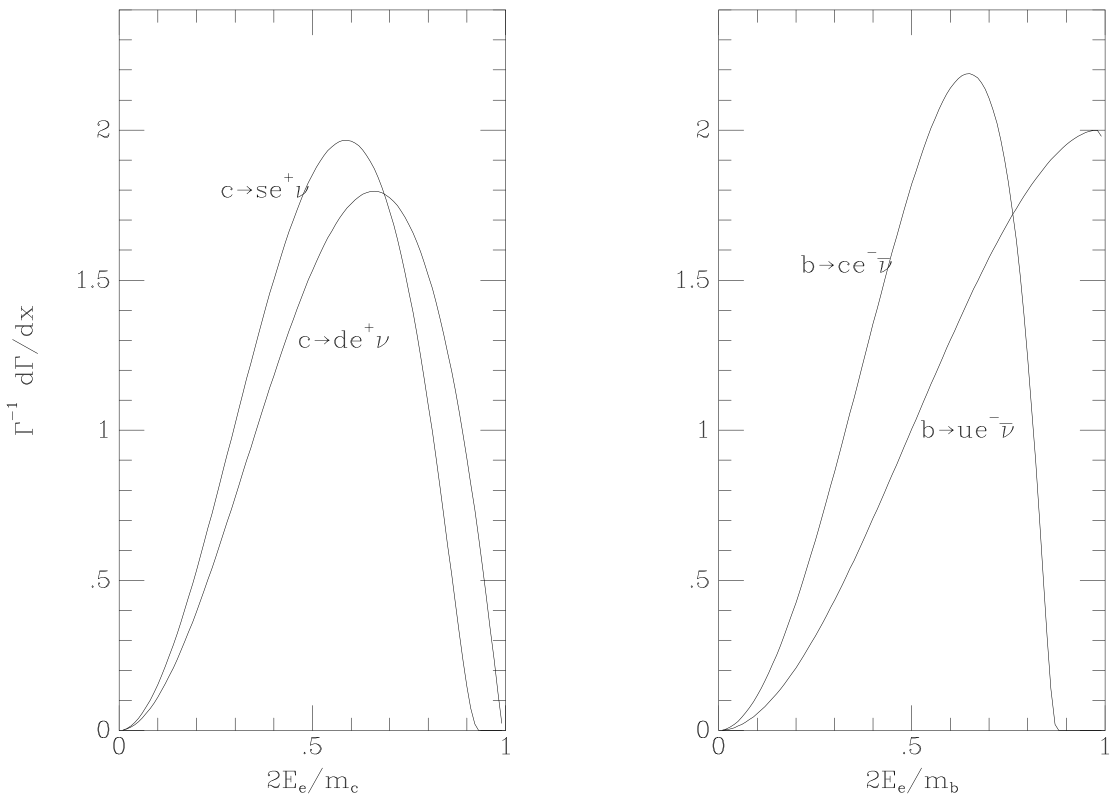
Γ_0 is the rescaled muon decay rate,

$$\Gamma_0(m_Q) = \frac{G_F^2 m_Q^5}{192\pi^3} \ .$$

Integrating over y the charged lepton spectra are

$$\frac{d\Gamma^{(c)}_{\textsf{sl}}}{dx} = |V_{cs}|^2 \Gamma_0(m_c) \left[\frac{12x^2(x_m - x)^2}{(1-x)} \right]$$

$$\frac{d\Gamma^{(b)}_{\textsf{sl}}}{dx} = |V_{cb}|^2 \Gamma_0(m_b) \left[\frac{2x^2(x_m - x)^2}{(1-x)^3} \right] (6 - 6x + x x_m + 2x^2 - 3x_m).$$



The e^+ from charm decay has a soft spectrum. The e^- from the CKM-disfavoured mode $b \to u$ has a hard spectrum.

- The measurement of leptons with energies beyond the kinematic limit for $b \to c$ gives information about V_{ub}.
- Allowing for theoretical uncertainty in endpoint region, the measured value is $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$

- After integration the semi-leptonic width including mass effects is found to be

$$\Gamma_{\text{sl}}^{(Q)} = |V_{Qq}|^2 \Gamma_0(m_Q) f\left(\frac{m_q}{m_Q}\right)$$

where the function f is given by

$$f(\epsilon) = (1-\epsilon^4)(1-8\epsilon^2+\epsilon^4) - 24\epsilon^4 \ln \epsilon\ .$$

Including the CKM-disfavoured mode $c \to d$ the result for the semi-leptonic decay of the c quark is

$$\Gamma^{(c)}_{\text{sl}} = \Gamma_0(m_c)\Big[f(m_s/m_c)|V_{cs}|^2 + f(m_d/m_c)|V_{cd}|^2\Big].$$

- For a rough estimate ignore strong interaction corrections and choose $m_c = 1.4\ \mathrm{GeV}$. The theoretical estimate for the semi-leptonic width is

$$\Gamma_{\text{sl}} = 1.1 \times 10^{-10}\ \mathrm{MeV}.$$

  From the measured semi-leptonic branching ratios of the D^+, $(17.2 \pm 1.9\%)$ and D^0, $(7.7 \pm 1.2\%)$ and the inverse of known lifetimes, we can calculate the semi-leptonic widths.

$$\begin{aligned}\Gamma_{\text{sl}}(D^0) &= (1.22 \pm 0.20) \times 10^{-10}\ \mathrm{MeV} \\ \Gamma_{\text{sl}}(D^+) &= (1.07 \pm 0.13) \times 10^{-10}\ \mathrm{MeV}.\end{aligned}$$

- The spectator model gives a fair description of the semi-leptonic decays of D mesons.

- For the semi-leptonic decays of B mesons, the theoretical decay width is

$$\Gamma^{(b)}_{\mathsf{sl}} = \Gamma_0(m_b) f\left(\frac{m_c}{m_b}\right)|V_{cb}|^2\eta_0,$$

- The CKM-disfavoured mode makes a negligible contribution to the total rate.
  $\eta_0 \approx 0.87$ due to strong interaction corrections.
  Using the measured semi-leptonic branching ratios of the $B^\pm$, $(10.1 \pm 1.8 \pm 1.5\%)$ and B^0, $(10.9 \pm 0.7 \pm 1.1\%)$, the semi-leptonic widths of the B mesons are

$$\begin{aligned} \Gamma_{\mathsf{sl}}(B^0) &= (0.48 \pm 0.12) \times 10^{-10}\ \mathrm{MeV} \\ \Gamma_{\mathsf{sl}}(B^+) &= (0.43 \pm 0.18) \times 10^{-10}\ \mathrm{MeV}. \end{aligned}$$

We can estimate V_{cb}. Choosing the values $m_b = 4.8\ \mathrm{GeV}, m_c = 1.4\ \mathrm{GeV}$ and $V_{cb} = 0.04$ one obtains

$$\Gamma_{\mathsf{sl}} = 2.7 \times 10^{-8}|V_{cb}|^2\ \mathrm{MeV} = 0.44 \times 10^{-10}\ \mathrm{MeV}\ .$$

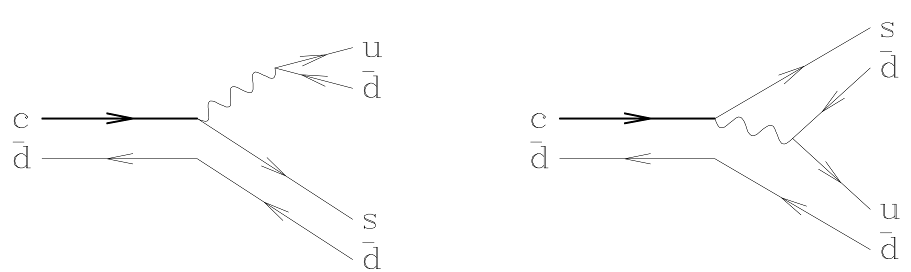
Hadronic decays

- Estimate for the total width using the spectator model. The width in the spectator model is given simply by the weak decay of the heavy quark followed by the subsequent decay of the resulting virtual W boson.
- Diagrams involving spectators are suppressed by powers of the heavy quark mass. For example, in the decay of a D_s^+ meson, a non-spectator diagram would result from the annihilation of the charm quark with the anti-strange quark.

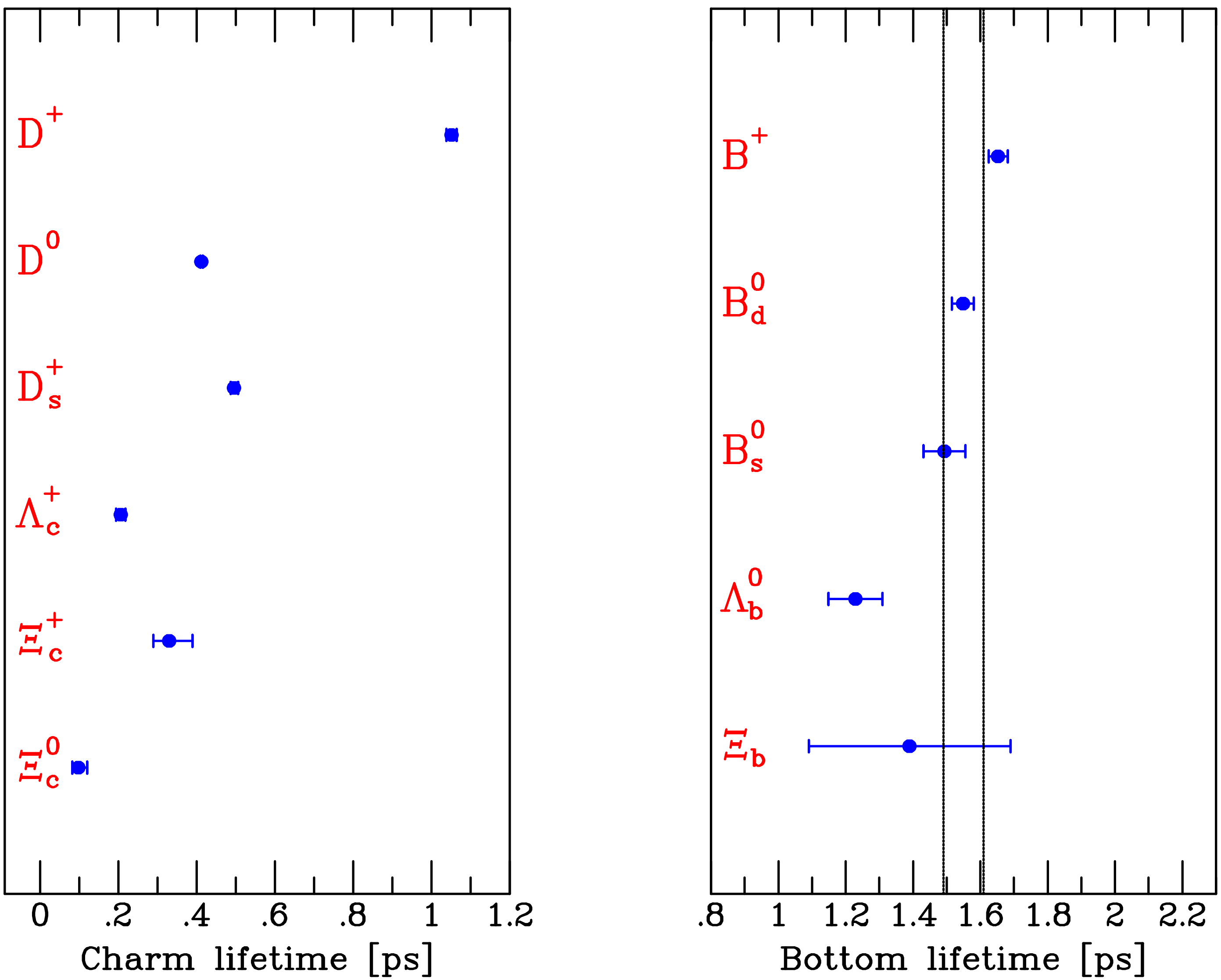
$$BR(c \to eX) = \frac{1}{1+1+3} \,,$$

ignoring strong interaction effects. Therefore the prediction of the simplest spectator model for the total width of a charmed hadron is given by multiplying the semi-leptonic width by five.

- This leads to an expected common lifetime for all charmed hadrons of the order of 1.2 ps. However the lifetimes of the D^+ and the D^0 mesons are very different.



Measured lifetimes



- Since the semi-leptonic widths are approximately equal, we can conclude that the failure of the spectator model is due to differences in the hadronic widths of the charmed hadrons.
- Reasons for the failure of the spectator model for D decays include non-spectator diagrams and strong radiative corrections.
- Interference effects should be suppressed by powers of m_c because of the small overlap of the $\bar{d}$ coming from the charm decay with the spectator $\bar{d}$. The former is initially localized in a volume of order $1/m_c^3$, whereas the latter is distributed throughout the D meson state.
- Since these interference effects appear to be important, we conclude that the charm quark is too light to be treated as a heavy quark in this context.

B total width

- Apply the spectator model to hadronic B decays.
- One might expect the B lifetime to be a factor of $(m_c/m_b)^5$ shorter than the estimate for the charm quark lifetime given above. However this mass effect is almost entirely cancelled by the factor of $|V_{cb}|^2$ which occurs in the expression for the width.
- The calculation of the semi-leptonic branching ratio for B decays is complicated even in the spectator model, because of the many channels which are kinematically allowed for the decay. A detailed calculation gives $BR(b \to eX) > 12.5\%$.
- The ground-state hadrons containing b quarks have roughly equal lifetimes.
- Calculating the total width from the theoretical semi-leptonic width and the measured semi-leptonic branching ratio, we obtain a lifetime of about 1.6 ps.
- The measured average b lifetime is 1.55 ± 0.06 ps
- This corresponds to a proper lifetime expressed in units of length of $c\tau = 463 \pm 18$ μm. A b quark with momentum 20 GeV has a relativistic γ factor of about 4. A B meson of this momentum decaying after one lifetime will travel about 1.9 mm. This decay distance large enough to be measured with a detector, typically a silicon vertex detector.

Top quark decays

- Standard Model. Since $m_t > M_W + m_b$ a top quark decays predominantly into a b quark and an on-shell W boson

$$t \rightarrow W^+ + b \quad \hookrightarrow l^+ + \nu$$

$$t \rightarrow W^+ + b \quad \hookrightarrow q + \bar{q}$$

- the branching ratio to leptons is given by counting the decay modes of the W, ($e\bar{\nu}_e$, $\mu\bar{\nu}_\mu$, $\tau\bar{\nu}_\tau$ and three colours of $u\bar{d}$ and $c\bar{s}$,

$$\mathrm{BR}(W^+ \rightarrow e^+\bar{\nu}) = \frac{1}{3+3+3} \approx 11\%.$$

With a perfect detector the numbers of events expected at $\sqrt{s} = 1.96$ TeV per fb^{-1} are

$$\begin{aligned} \text{N}(e\mu\ \text{+jets}) &= 2 \times .11 \times .11 \times 7500 \approx 180 \\ \text{N}(e\ +\ \text{jets}) &= 2 \times .11 \times .66 \times 7500 \approx 1100. \end{aligned}$$

The existence of both of these decay modes with the correct ratio is a first test of the decay modes of the top.

The W boson coming from top decay can be either left-handed (L) or longitudinally (0) polarized.

$$\overline{\sum}|\mathcal{M}_L|^2 = \frac{2G_F m_t^4}{\sqrt{2}}|V_{tb}|^2\left[2x^2(1-x^2+y^2)\right]$$

$$\overline{\sum}|\mathcal{M}_0|^2 = \frac{2G_F m_t^4}{\sqrt{2}}|V_{tb}|^2\left[1-x^2-y^2(2+x^2-y^2)\right],$$

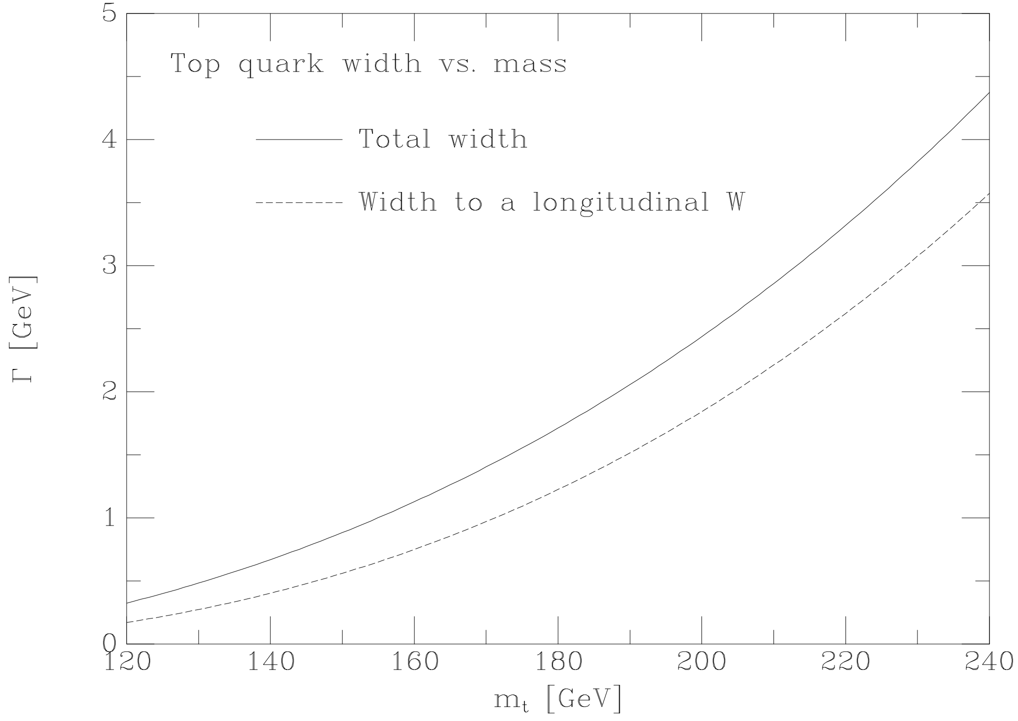
where $x = M_W/m_t, y = m_b/m_t$.
In the limit $m_t \gg M_W$ the result for the total width is

$$\Gamma(t \to bW) = \frac{G_F m_t^3}{8\pi\sqrt{2}}|V_{tb}|^2 \approx 1.76\ \mathrm{GeV}\left(\frac{m_t}{175\ \mathrm{GeV}}\right)^3.$$

$V_{tb} \approx 1$ as suggested by the unitarity relation

$$|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1.$$

- The top quark has a ‘semi-weak’ decay rate.
- The lifetime of the top quark is only of order 10^{-25} seconds ($c\tau \sim 10^{-10}\mu$m) and it therefore decays before it has time to hadronize.



- The polarization state of the W controls the angular distribution of the leptons into which it decays. We may define the lepton helicity angle θ_e^*, which is the angle of the charged lepton in the rest frame of the W, with respect to the original direction of travel of the W (*i.e.* anti-parallel to the recoiling b quark). If the b quark jet is identified, this angle can be defined experimentally as

$$\cos\theta_e^* \approx \frac{b\cdot(e^+ - \nu)}{b\cdot(e^+ + \nu)} \approx \frac{4b\cdot e^+}{m_t^2 - M_W^2} - 1,$$

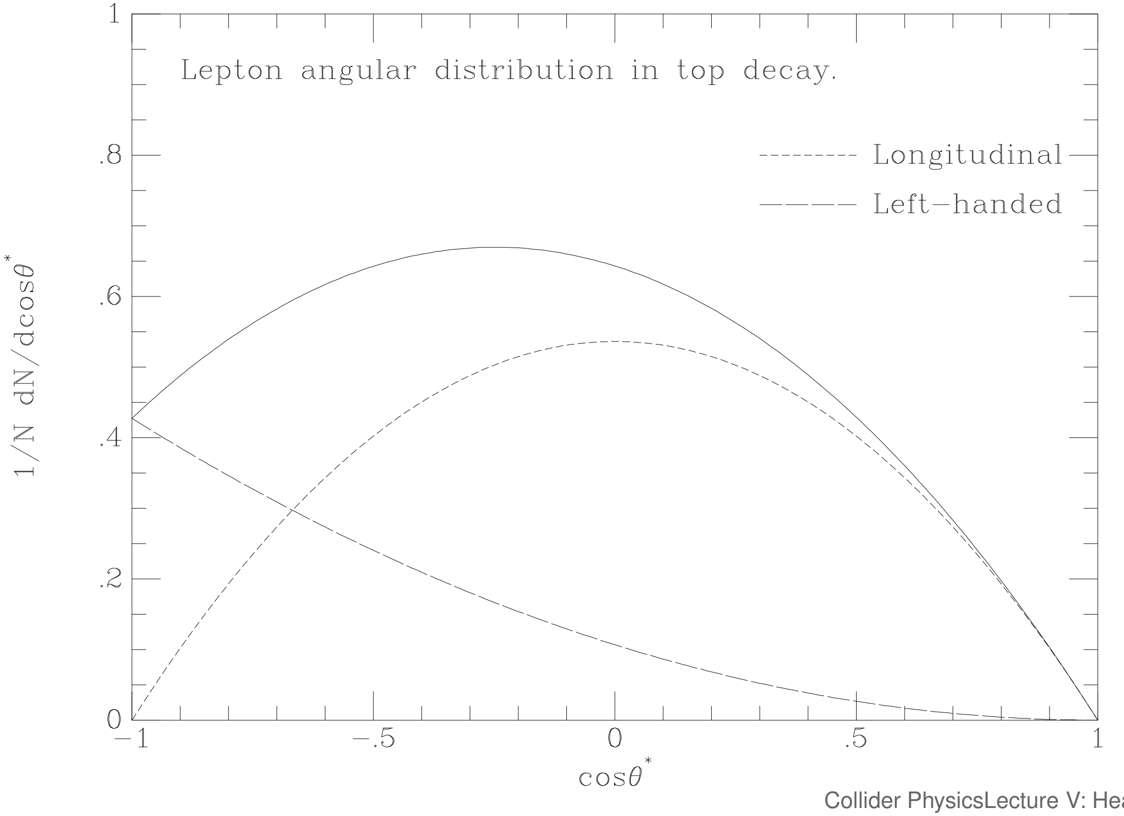
in an obvious notation where t, b, e^+ and ν represent four-momenta.

$$\begin{aligned}\overline{\sum}|\mathcal{M}^{(t)}|^2 \quad &= \quad \left[\overline{\sum}|\mathcal{M}_0|^2 \times |D_0|^2 + \overline{\sum}|\mathcal{M}_L|^2 \times |D_L|^2\right] \\ &\times \frac{\pi}{M_W\Gamma_W}\,\delta(w^2 - M_W^2).\end{aligned}$$

Here M_0, M_L are given above with $y = 0$ and D_0, D_L are the helicity amplitudes for the decay of a longitudinal and left-handed W boson respectively:

$$|D_0|^2 = \frac{G_F M_W^4}{\sqrt{2}} \frac{1}{2} \sin^2 \theta_e^*$$

$$|D_L|^2 = \frac{G_F M_W^4}{\sqrt{2}} \frac{1}{4} (1 - \cos \theta_e^*)^2.$$

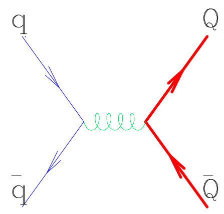


Heavy quark production, leading order

The leading-order processes for the production of a heavy quark Q of mass m in hadron-hadron collisions

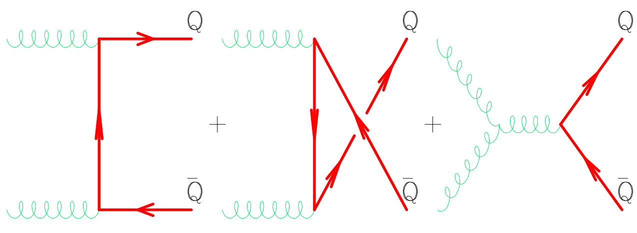
$$\begin{array}{ll} (a) & q(p_1) + \overline{q}(p_2) \to Q(p_3) + \overline{Q}(p_4) \\ (b) & g(p_1) + g(p_2) \to Q(p_3) + \overline{Q}(p_4) \end{array}$$

where the four-momenta of the partons are given in brackets.



(a)

Process	$\overline{\sum}\lvert\mathcal{M}\rvert^2/g^4$
$q\,\overline{q} \to Q\,\overline{Q}$	$\frac{4}{9}\left(\tau_1^2+\tau_2^2+\frac{\rho}{2}\right)$
$g\,g \to Q\,\overline{Q}$	$\left(\frac{1}{6\tau_1\tau_2}-\frac{3}{8}\right)\left(\tau_1^2+\tau_2^2+\rho-\frac{\rho^2}{4\tau_1\tau_2}\right)$



(b)

$\overline{\sum}$ indicates averaged (summed) over initial (final) colours and spins

We have introduced the following notation for the ratios of scalar products:

$$\tau_1 = \frac{2p_1.p_3}{\hat{s}}, \quad \tau_2 = \frac{2p_2.p_3}{\hat{s}}, \quad \rho = \frac{4m^2}{\hat{s}}, \quad \hat{s} = (p_1 + p_2)^2.$$

- The short-distance cross section is obtained from the invariant matrix element in the usual way:

$$d\hat{\sigma}_{ij} = \frac{1}{2\hat{s}} \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

  The first factor is the flux factor for massless incoming particles. The other terms come from the phase space for $2 \to 2$ scattering.

- In terms of the rapidity $y = \frac{1}{2} \ln((E + p_z)/(E - p_z))$ and transverse momentum, p_T, the relativistically invariant phase space volume element of the final-state heavy quarks is

$$\frac{d^3p}{E} = dy \; d^2p_T \; .$$

The result for the invariant cross section may be written as

$$\frac{d\sigma}{dy_3 dy_4 d^2p_T} = \frac{1}{16\pi^2 \hat{s}^2} \sum_{ij} x_1 f_i(x_1,\mu^2)\; x_2 f_j(x_2,\mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

x_1 and x_2 are fixed if we know the transverse momenta and rapidity of the outgoing heavy quarks. In the centre-of-mass system of the incoming hadrons we may write

$$\begin{aligned}
p_1 &= \frac{1}{2}\sqrt{s}(x_1, 0, 0, x_1) \\
p_2 &= \frac{1}{2}\sqrt{s}(x_2, 0, 0, -x_2) \\
p_3 &= (m_T \cosh y_3, p_T, 0, m_T \sinh y_3) \\
p_4 &= (m_T \cosh y_4, -p_T, 0, m_T \sinh y_4).
\end{aligned}$$

Applying energy and momentum conservation, we obtain

$$x_1 = \frac{m_T}{\sqrt{s}}\left(e^{y_3} + e^{y_4}\right), x_2 = \frac{m_T}{\sqrt{s}}\left(e^{-y_3} + e^{-y_4}\right), \hat{s} = 2m_T^2(1 + \cosh \Delta y).$$

The quantity $m_T = \surd(m^2 + p_T^2)$ is the transverse mass of the heavy quarks and $\Delta y = y_3 - y_4$ is the rapidity difference between them.

In these variables the leading order cross section is

$$\frac{d\sigma}{dy_3 dy_4 d^2 p_T} = \frac{1}{64\pi^2 m_T^4 (1+\cosh(\Delta y))^2} \sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2)\, \overline{\sum} |\mathcal{M}_{ij}|^2 .$$

Expressed in terms of m, m_T and Δy, the matrix elements for the two processes are

$$\overline{\sum} |\mathcal{M}_{q\bar{q}}|^2 = \frac{4g^4}{9} \Big(\frac{1}{1+\cosh(\Delta y)} \Big) \Big(\cosh(\Delta y) + \frac{m^2}{m_T^2} \Big) ,$$

$$\overline{\sum} |\mathcal{M}_{gg}|^2 = \frac{g^4}{24} \Big(\frac{8\cosh(\Delta y) - 1}{1+\cosh(\Delta y)} \Big) \Big(\cosh(\Delta y) + 2\frac{m^2}{m_T^2} - 2\frac{m^4}{m_T^4} \Big).$$

- As the rapidity separation Δy between the two heavy quarks becomes large

$$\overline{\sum} |\mathcal{M}_{q\bar{q}}|^2 \sim \text{ constant}, \quad \overline{\sum} |\mathcal{M}_{gg}|^2 \sim \exp \Delta y \; .$$

- The cross section is damped at large Δy and heavy quarks produced by $q\bar{q}$ annihilation are more closely correlated in rapidity those produced by gg fusion.

Applicability of perturbation theory?

- Consider the propagators in the diagrams.

$$
\begin{aligned}
(p_1+p_2)^2 = &\quad 2p_1.p_2 &= 2m_T^2\big(1+\cosh\Delta y\big)\ ,\\
(p_1-p_3)^2 - m^2 = &\quad -2p_1.p_3 &= -m_T^2\big(1+e^{-\Delta y}\big)\ ,\\
(p_2-p_3)^2 - m^2 = &\quad -2p_2.p_3 &= -m_T^2\big(1+e^{\Delta y}\big)\ .
\end{aligned}
$$

  Note that the propagators are all off-shell by a quantity of least of order m^2.

- Thus for a sufficiently heavy quark we expect the methods of perturbation theory to be applicable. It is the mass m (which by supposition is very much larger than the scale of the strong interactions Λ) which provides the large scale in heavy quark production. We expect corrections of order Λ/m
- This does not address the issue of whether the charm or bottom mass is large enough to be adequately described by perturbation theory.

Heavy quark production in $O(\alpha_S^3)$

In NLO heavy quark production m is the heavy quark mass.

$$\sigma(S) = \sum_{i,j} \int \, dx_1 dx_2 \; \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) F_i(x_1, \mu^2) F_j(x_2, \mu^2)$$

$$\hat{\sigma}_{i,j}(\hat{s}, m^2, \mu^2) = \sigma_0 c_{ij}(\hat{\rho}, \mu^2)$$

where $\hat{\rho} = 4m^2/\hat{s}, \bar{\mu}^2 = \mu^2/m^2, \sigma_0 = \alpha_S^2(\mu^2)/m^2$ and $\hat{s}$ in the parton total c-of-m energy squared. The coupling satisfies

$$\frac{d\alpha_S}{d\ln\mu^2} = -b_0 \frac{\alpha_S^2}{2\pi} + O(\alpha_S^3), \;\; b_0 = \frac{11N - 2n_f}{6}$$

$$c_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = c_{ij}^{(0)}(\rho) + 4\pi\alpha_S(\mu^2)\left[c_{ij}^{(1)}(\rho) + \overline{c}_{ij}^{(1)}(\rho)\ln(\frac{\mu^2}{m^2})\right] + O(\alpha_S^2)$$

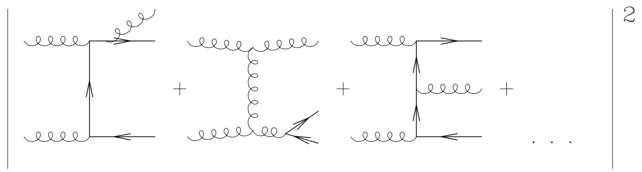
The lowest-order functions $c_{ij}^{(0)}$ are obtained by integrating the lowest order matrix elements

$$
\begin{aligned}
c_{q\overline{q}}^{(0)}(\rho) &= \frac{\pi\beta\rho}{27}\left[(2+\rho)\right] ,\\
c_{gg}^{(0)}(\rho) &= \frac{\pi\beta\rho}{192}\left[\frac{1}{\beta}\left[\rho^2+16\rho+16\right]\ln\left(\frac{1+\beta}{1-\beta}\right)-28-31\rho\right] ,\\
c_{gq}^{(0)}(\rho) &= c_{g\overline{q}}^{(0)}(\rho) = 0 ,
\end{aligned}
$$

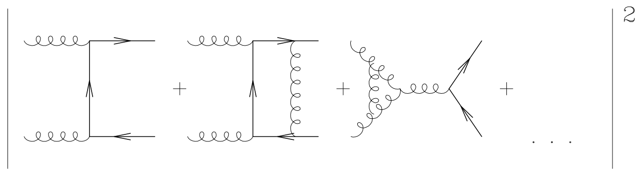
and $\beta = \sqrt{1-\rho}$.

- The functions $c_{ij}^{(0)}$ vanish both at threshold ($\beta \to 0$) and at high energy ($\rho \to 0$).
- Note that the quark-gluon process is zero in lowest order, but is present in higher orders.

- The functions $c_{ij}^{(1)}$ are also known



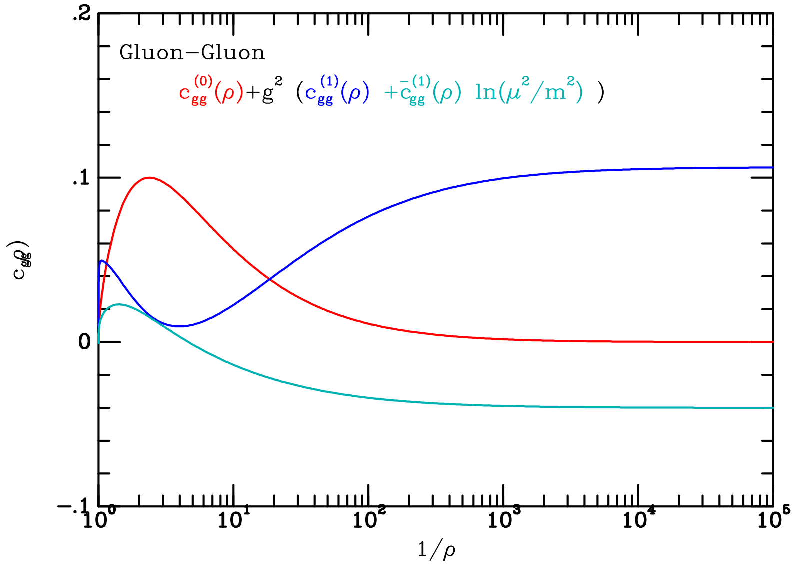
Real emission diagrams

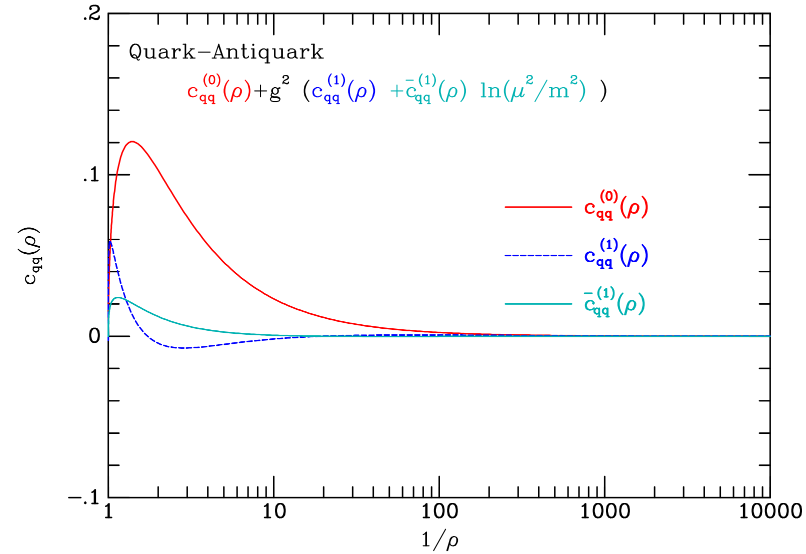


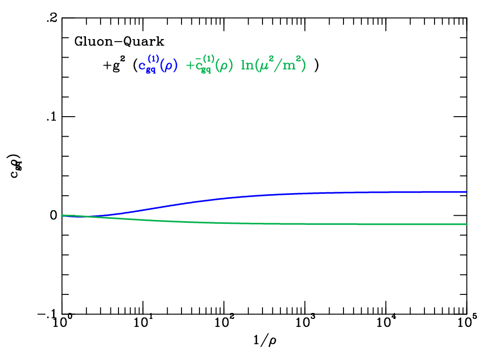
Virtual emission diagrams

- Examples of higher-order corrections to heavy quark production.
- In order to calculate the c_{ij} in perturbation theory we must perform both renormalization and factorization of mass singularities. The subtractions required for renormalization and factorization are done at mass scale μ.

Higher order results, $c_{ij}^{(1)}$







μ dependence

μ is an unphysical parameter. The physical predictions should be invariant under changes of μ at the appropriate order in perturbation theory. If we have performed a calculation to $O(\alpha_S^3)$, variations of the scale μ will lead to corrections of $O(\alpha_S^4)$,

$$\mu^2 \frac{d}{d\mu^2} \sigma = O(\alpha_S^4).$$

- The term $\overline{c}^{(1)}$, which controls the μ dependence of the higher-order perturbative contributions, is fixed in terms of the lower-order result $c^{(0)}$:

$$\begin{aligned}\overline{c}_{ij}^{(1)}(\rho) &= \frac{1}{8\pi^2}\Bigg[4\pi b c_{ij}^{(0)}(\rho) - \int_\rho^1 dz_1 \sum_k c_{kj}^{(0)}(\frac{\rho}{z_1}) P_{ki}^{(0)}(z_1) \\ &\quad - \int_\rho^1 dz_2 \sum_k c_{ik}^{(0)}(\frac{\rho}{z_2}) P_{kj}^{(0)}(z_2)\Bigg].\end{aligned}$$

In obtaining this result we have used the renormalization group equation for the running coupling

$$\mu^2 \frac{d}{d\mu^2} \alpha_S(\mu^2) = -b\alpha_S^2 + \dots$$

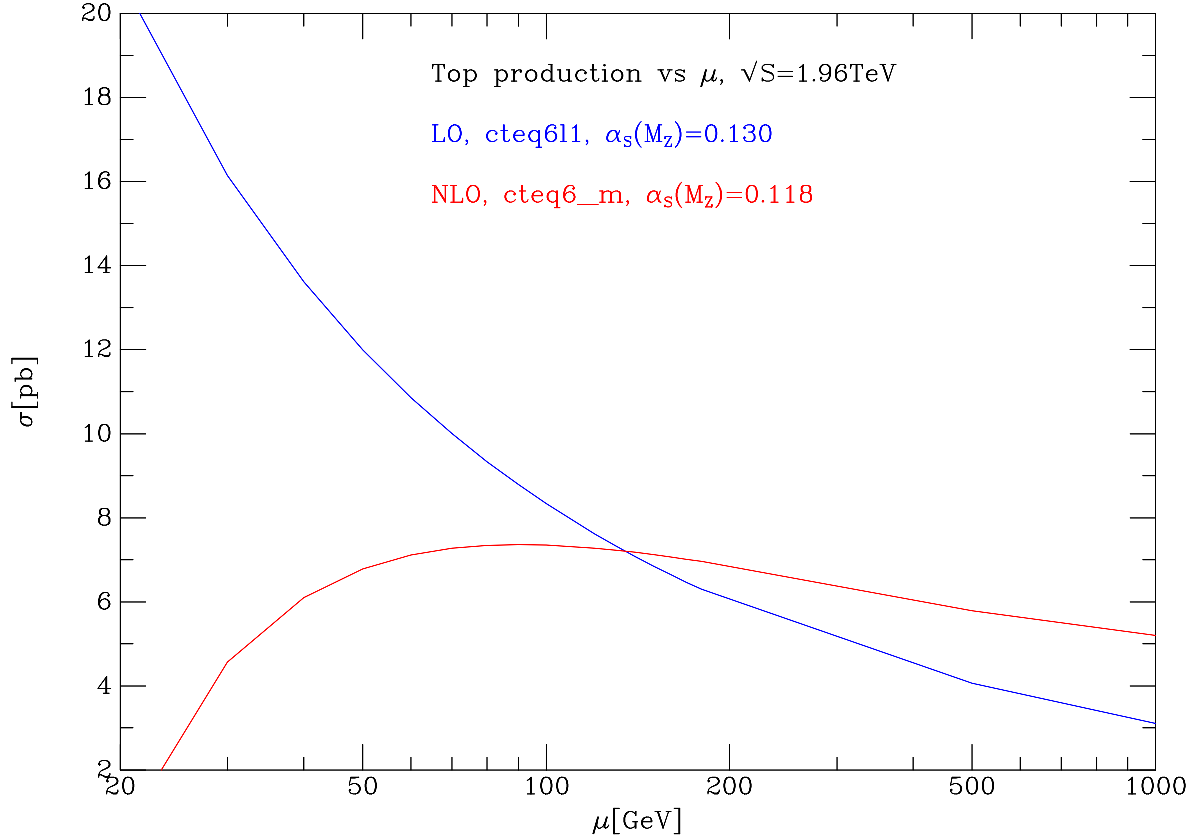
and the lowest-order form of the GLAP equation

$$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \sum_k \int_x^1 \frac{dz}{z} P_{ik}^{(0)}(z) f_k(\frac{x}{z},\mu^2) + \dots .$$

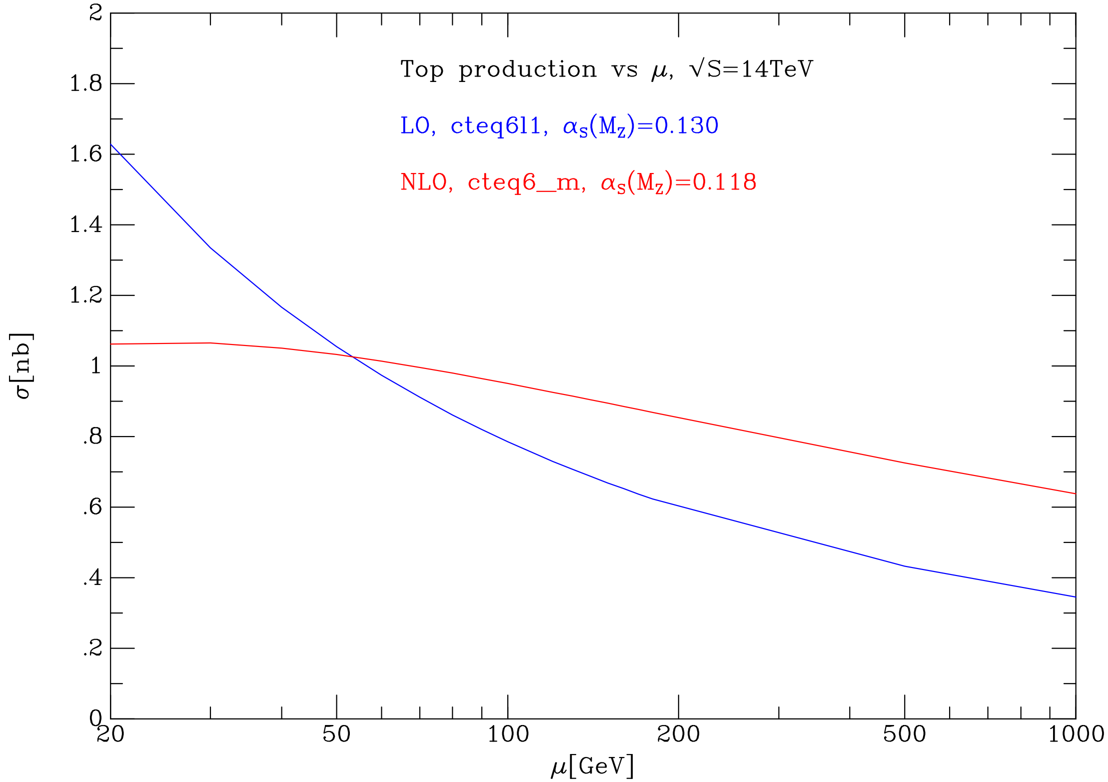
This illustrates an important point which is a general feature of renormalization group improved perturbation series in QCD. The coefficient of the perturbative correction depends on the choice made for the scale μ, but the scale dependence changes the result in such a way that the physical result is independent of that choice. Thus the scale dependence is formally small because it is of higher order in α_S. This does not assure us that the scale dependence is actually *numerically* small for all series. A pronounced dependence on the scale μ is a signal of an untrustworthy perturbation series.

Scale dependence in top production

- Inclusion of the higher order terms leads to a stabilization of the top cross section.
- No unequivocal method for estimating the theoretical error, but it is clear that it it smaller at NLO.



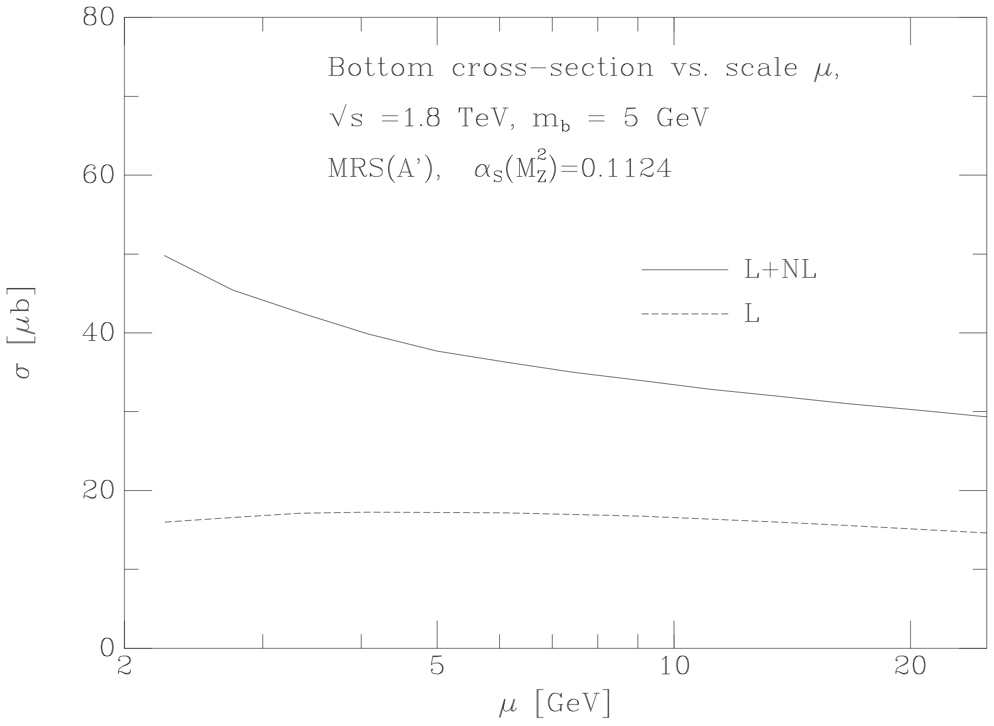
Top production at LHC



- At LHC top cross section is more than 100 times bigger than at Tevatron.
- Maximum at a rather low scale, $\mu \sim 30\mathrm{GeV}$.

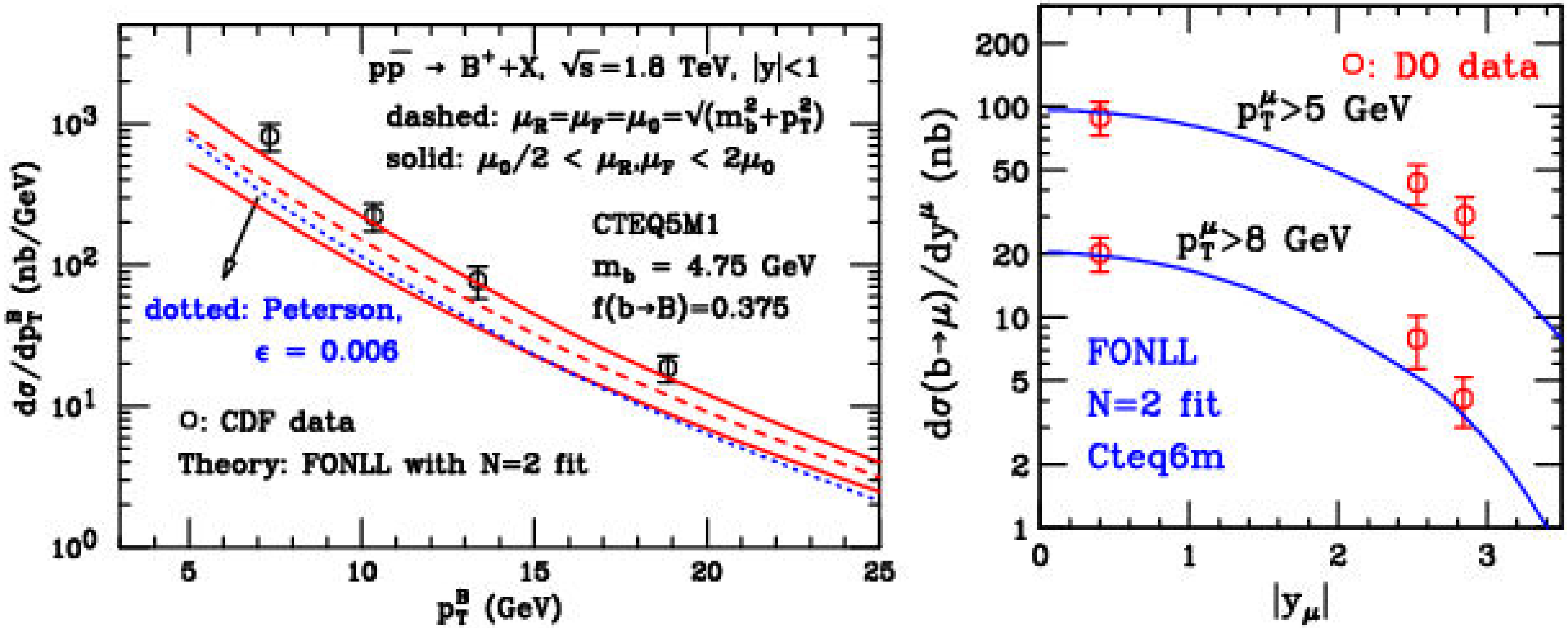
Scale dependence in bottom production

- The perturbation series for bottom quark production is not well behaved.
- The lowest order cross section is almost μ independent because of an accidental cancellation between the fall-off of α_S and the increase of the gluon distributions with increasing μ.



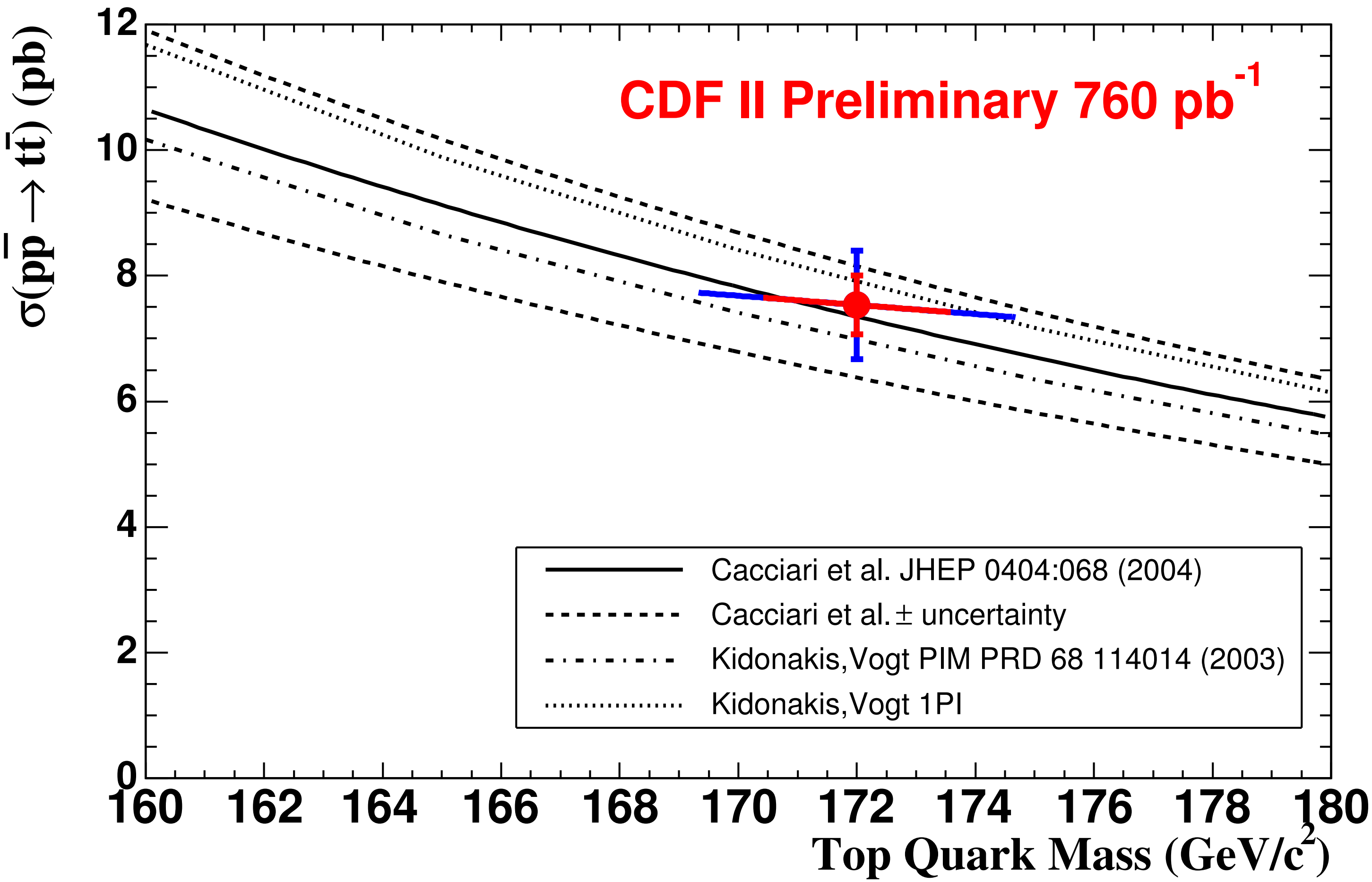
Beauty production at CDF

Nason et al.

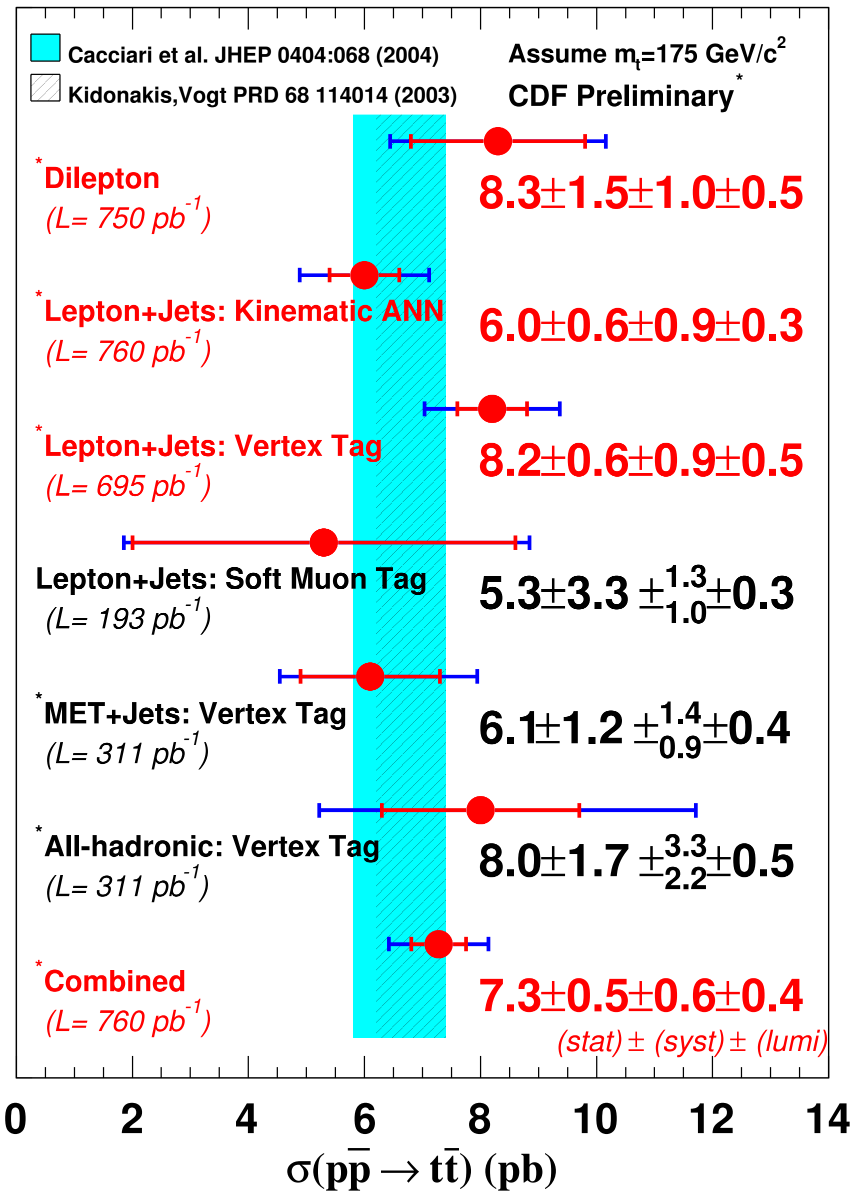


- To compare the b-quark production prediction with the experimental data on B-meson production, we need to include a fragmentation function.
- This comparison was flawed in the past, because inadequate fragmentation functions were used, see hep-ph/0411020
- The data agrees with the upper range of the theoretical prediction. Given the status of the perturbation theory for b-quark production, this is a positive outcome.

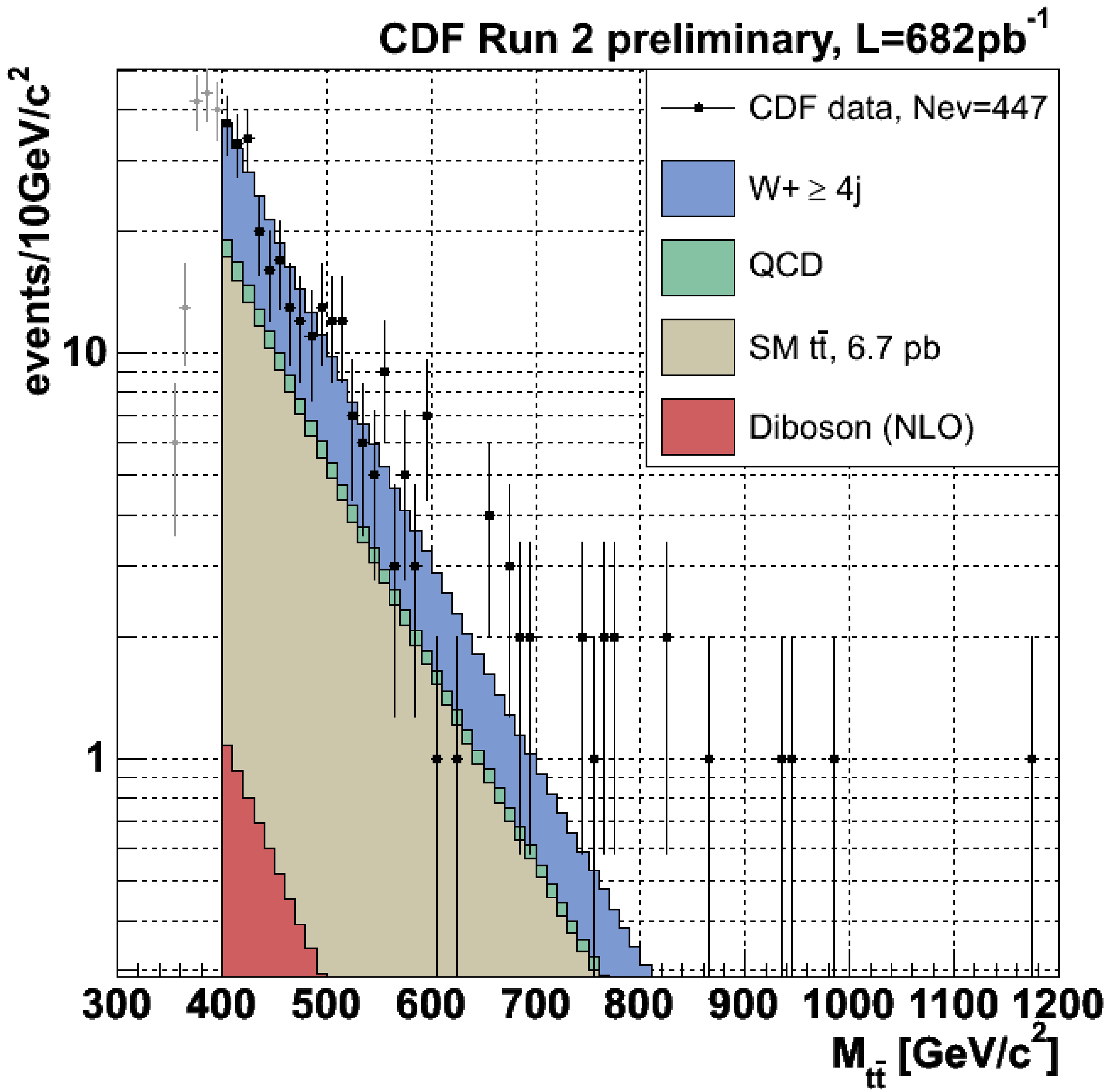
Top production cross section vs. Theory



Top production cross section vs. Theory



Distribution in $t\bar{t}$ invariant mass



Top production

- All the information on the top quark is still rather limited and crude
- Within errors agreement between three generation theory and experiment

	Tevatron 2006 Results	Theory	Lum.[fb^{-1}]
m_t	172.5 ± 2.5 GeV	178.9^{+12}_{-9}GeV	0.6-0.8
W-helicity	$f_0 = 0.74(+0.22)(-0.34)$	$\simeq 0.7$	0.16
	$f_+ = 0.08 \pm 0.10$	$\simeq 0$	0.37
Charge	rule out Q = +4/3 (93.7% CL)	2/3	0.23
lifetime	$c\tau < 53\ \mu m$ (95% CL)	$\simeq 10^{-10} \mu m$	0.32
V_{tb}	awaits single top discovery	0.9990-0.9993	>1.5?
BR($t \to Wb$)/BR($t \to Wq$)	1.03 (+0.19)(-0.17)	1	0.23
Resonances $X_0 \to t\bar{t}$	$M(X_0) > 725$ GeV (95% CL)	-	0.68
4th Generation t' quark	$m(t') > 258$ GeV (95% CL)	-	0.76

The CKM matrix

The primed down-type quarks q' differ from the mass eigenstate combinations q by a unitary transformation, V:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} .$$

V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

The constraint of unitarity is $V^\dagger V = 1$. Written out explicitly, this imposes nine conditions

$$\sum_{j=1}^{3} |V_{ij}|^2 = \sum_{i=1}^{3} |V_{ij}|^2 = 1 \ ,$$

$$\sum_{k=1}^{3} V_{ki}^* V_{kj} = 0 \quad (i \neq j) \ .$$

CKM experiment

The experimental limits on the magnitudes of individual matrix elements, taking into account constraints derived from unitarity, are

$$V = \begin{pmatrix} 0.9742-0.9757 & 0.219-0.226 & 0.002-0.005 \\ 0.219-0.225 & 0.9734-0.9749 & 0.037-0.043 \\ 0.004-0.014 & 0.035-0.043 & 0.9990-0.9993 \end{pmatrix}.$$

If we do not assume three generation unitarity,

$$V = \begin{pmatrix} 0.9722-0.9748 & 0.216-0.223 & 0.002-0.005 & \ldots \\ 0.209-0.228 & 0.959-0.976 & 0.037-0.043 & \ldots \\ 0-0.09 & 0-0.16 & 0.07-0.993 & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}.$$

Single Top production

- D0 has measured

$$\frac{BR(t \to Wb)}{BR(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 1.03^{+0.19}_{-0.17}$$

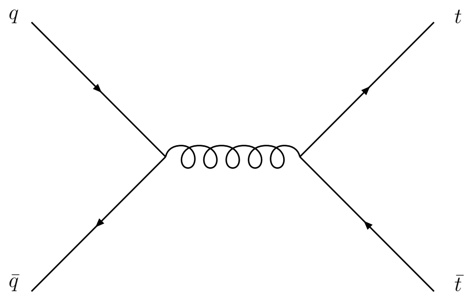
  If we assume just three generations of quarks, unitarity of the CKM matrix implies that the denominator is equal to one, so that we can extract

$$|V_{tb}| = 1.01 \pm 0.09$$

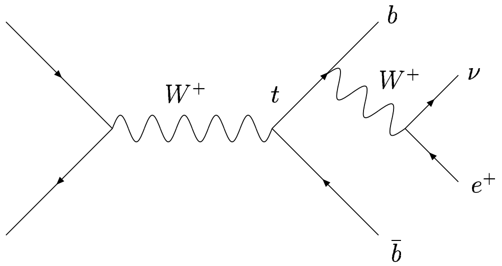
- But assuming unitarity we know already that $V_{tb} = 0.9990 - 0.9993$.
- The current D0 measurement shows that $|V_{tb}| \gg |V_{td}|, |V_{ts}|$.
- For a real measurement of V_{tb} we must look at the electroweak production of a top quark.

Producing the top quark

- The top quark was discovered in Run I of the Tevatron by producing it in pairs:

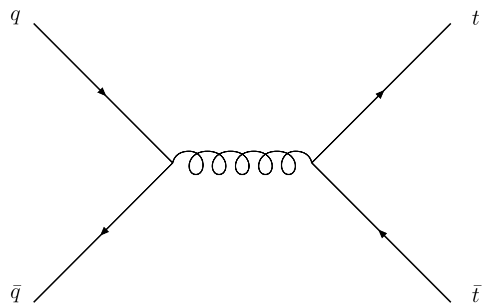


- However, it should also be possible to produce it singly in Run II, for example:

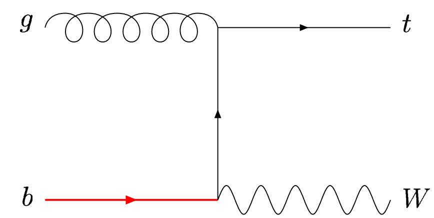


- This is especially interesting since it would yield information about the weak interaction of top quarks (V_{tb}).

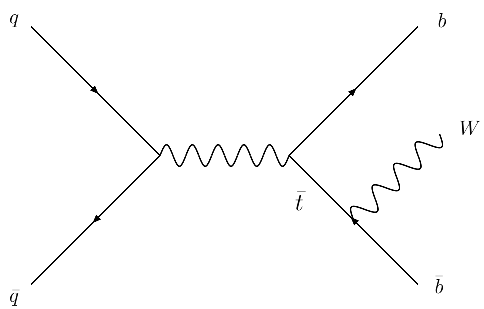
Top production rates



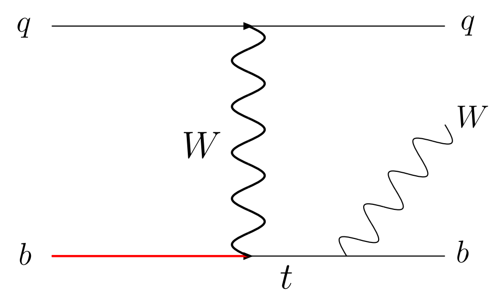
6 pb
720 pb



0.14 pb
66 pb



0.8 pb
10 pb

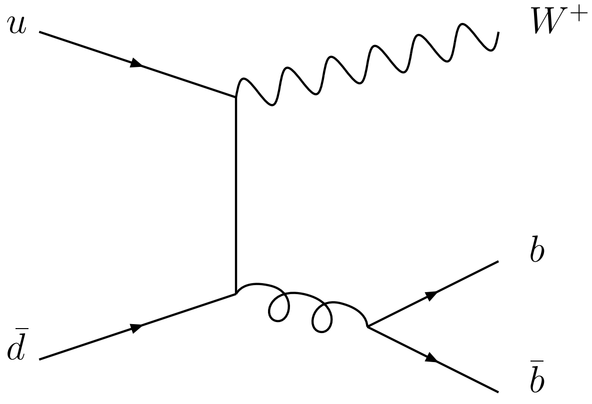


1.8 pb
240 pb

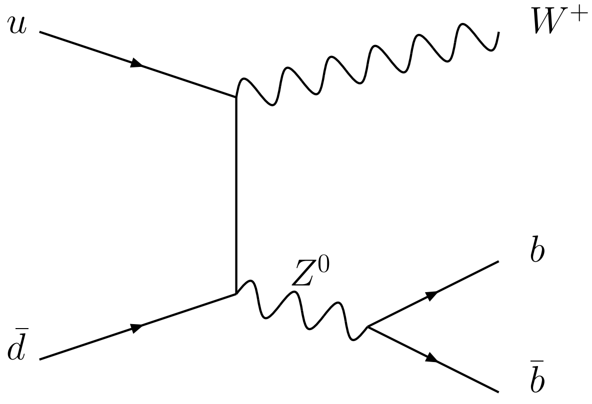
- All cross-sections are known to NLO (Tevatron / LHC)
- The total single top cross-section is smaller than the $t\bar{t}$ rate by about a factor of two, at both machines

Experimental signature

- The experimental “signature” is an event which contains a top quark – identified by the combined mass of its decay products – and which also has two jets containing b-quarks. These can be distinguished from other jets around 50% of the time.
- Observed events such as these can also be the result of other basic processes. These backgrounds include, for example:



$Wb\bar{b}$

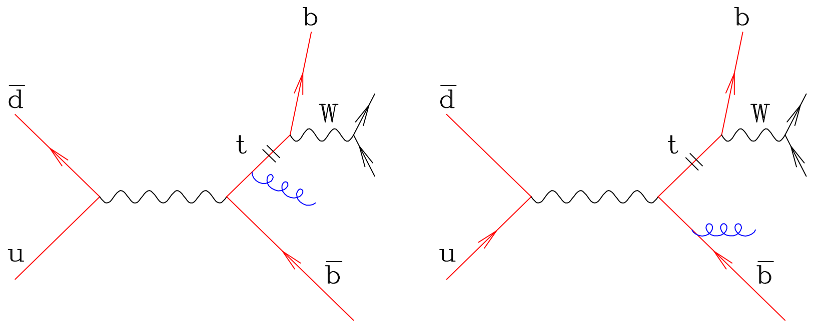


$WZ(\rightarrow b\bar{b})$

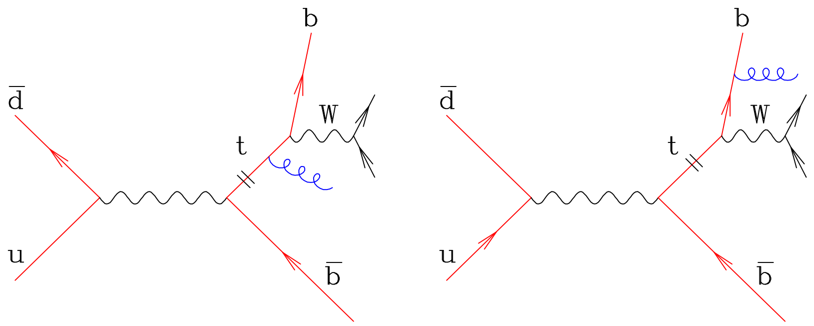
- MCFM can calculate the signal and backgrounds at NLO.

Inclusion of decay

- Results had previously been presented without including the decay of the top quark. Without it, predictions for some quantities used in Tevatron search strategies are impossible
- Final state radiation that enters at next-to-leading order is possible in either the production or decay phase:

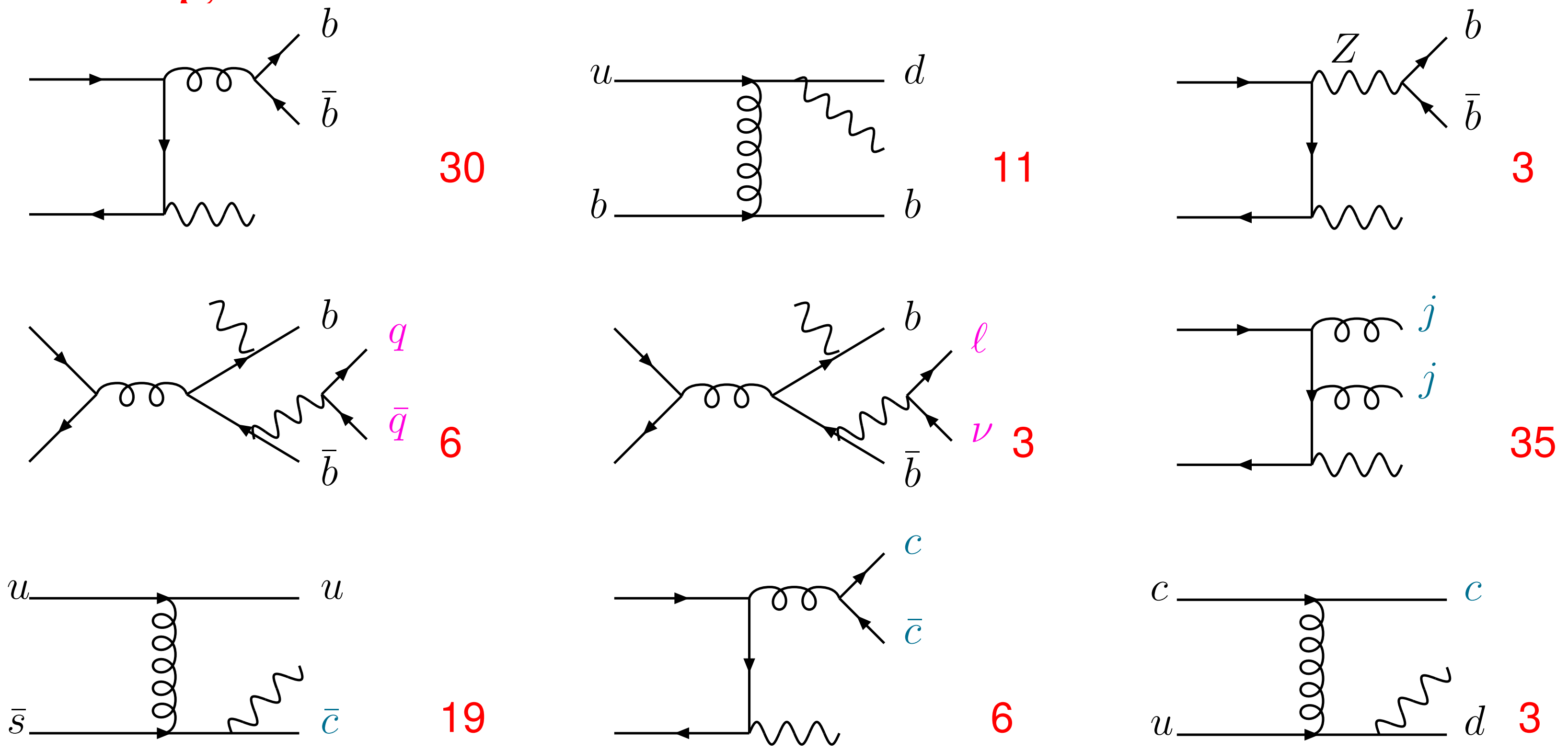


production



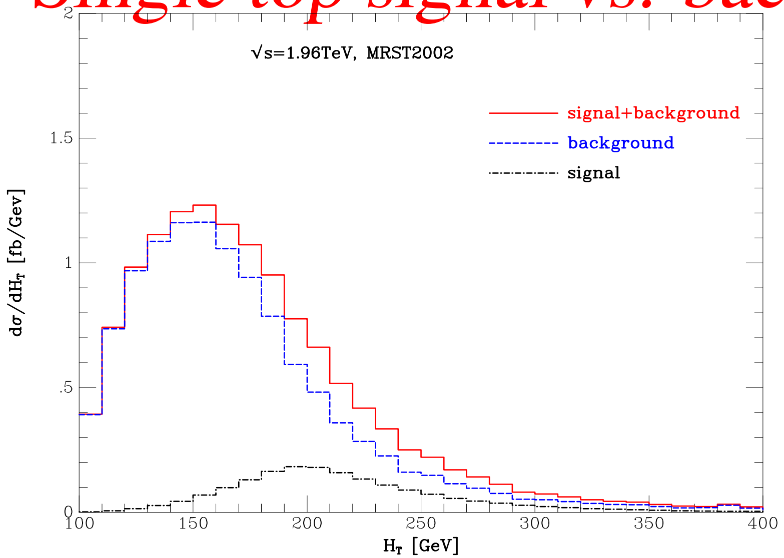
decay

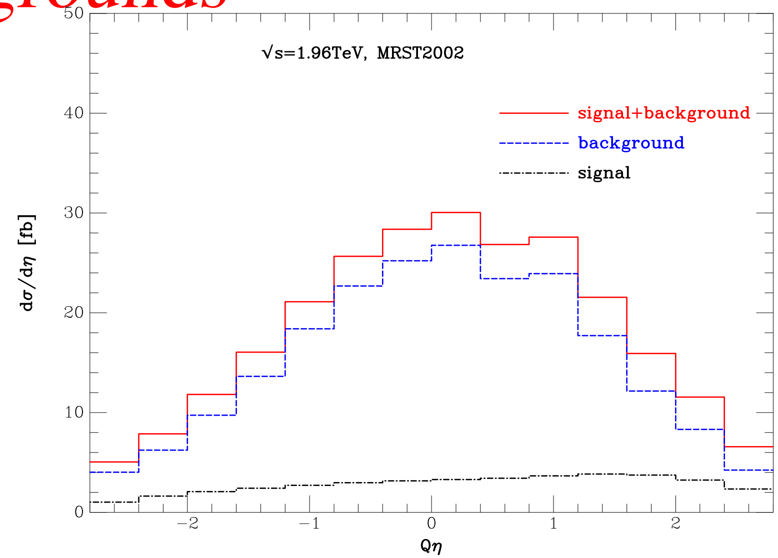
Backgrounds



- Cross-sections in fb include nominal tagging efficiences and mis-tagging/fake rates. Calculated with MCFM, most at NLO
- Rates are 7 fb and 11 fb for s- and $t-$channel signal

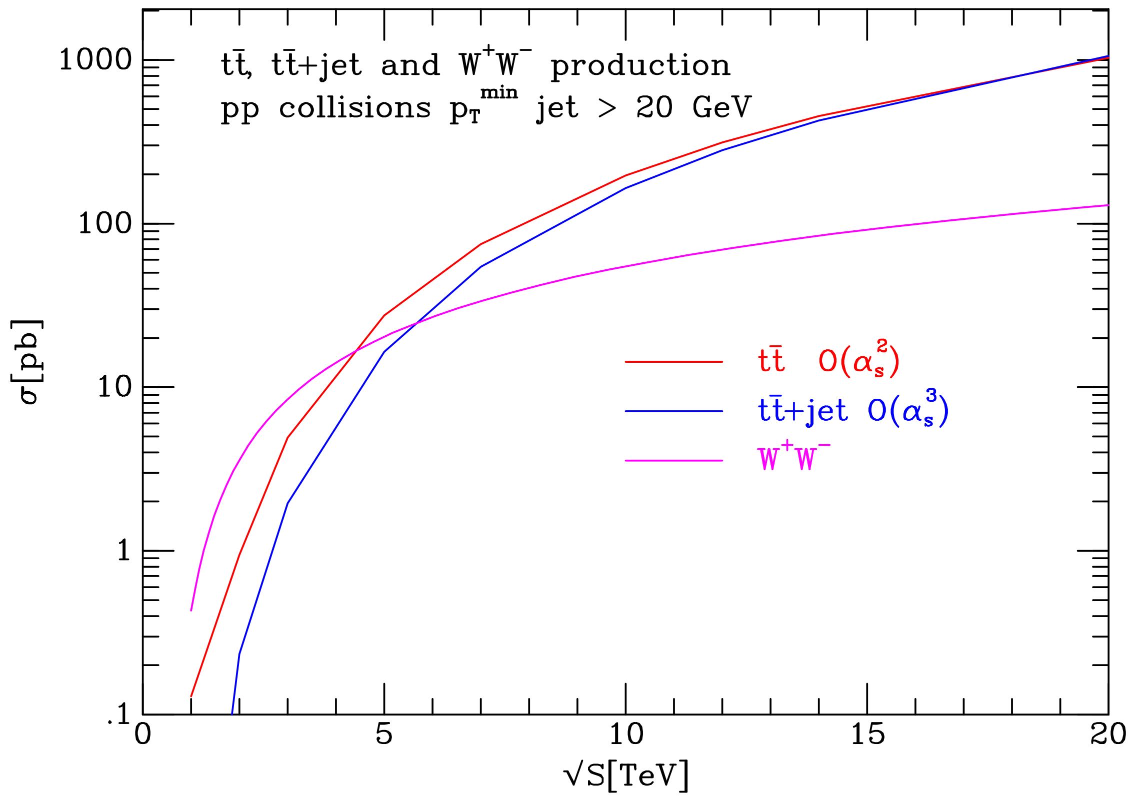
Single top signal vs. backgrounds





- $H_T =$ scalar sum of jet, lepton and missing E_T
- $Q\eta$ is the product of the lepton charge and the rapidity of the untagged jet, useful for picking out the t-channel process
- Signal:Background (with our nominal efficiencies) is about $1 : 6$
  – a very challenging measurement indeed. Production in this mode has not yet been observed at Fermilab.
- it will take 1.5 fb^{-1} to have evidence (3σ) for single top from a single experiment at the Tevatron (Gresele, Moriond 2006).

Top +jet production at LHC

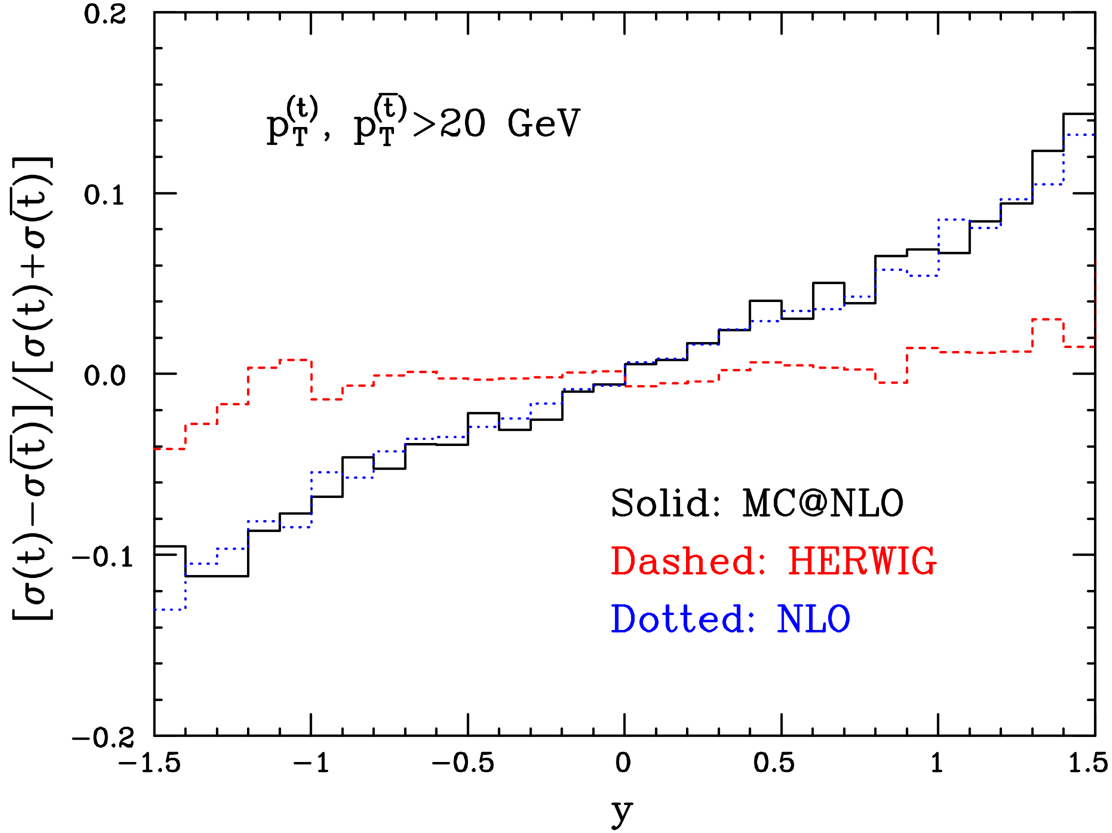


- $t\bar{t}$+jet cross section same as $t\bar{t}$ cross section; Radiation probability is one.
- Note that a $p_T = 20$ GeV jet can be adequately described using the soft approximation.
- The W^+W^- cross section is also shown, (subject to gauge cancellation)

Asymmetry in top production

Frixione,Nason,Webber

- A further improvement of the theory of $t\bar{t}$ production is the combination of NLO calculations with shower Monte Carlos.
- Example of $t\bar{t}$-production using MC@NLO
- NLO curve (in blue, dotted).



Recap

- Simple spectator model gives a poor description of hadronic charm decays and a good description of bottom decay.
- Top decay is a semi-weak process
- Heavy quark production is calculable in perturbation theory inasmuch as m_Q is greater than Λ QCD.
- Bottom quark production gives a troubled perturbation series, but with care on the fragmentation, a fair agreement can be found.
- Top quark production at the Tevatron is in good agreement with theory.
- Single top has a sizeable cross section, but remains elusive because it plagued by many backgrounds.
- The top quark is produced at LHC at a large rate and with many jets. Both parton shower and NLO cross-sction and included in MC@NLO.